Rotating Machinery

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Nomenclature

- Ω rotor rotative speed
- K rotor system modal stiffness
- M rotor system modal mass
- D rotor system modal damping
- λ fluid average circumferential velocity ratio
- d displacement
- v velocity
- a acceleration

Displacement, velocity, and acceleration

Before discussing the nature of Bode and polar plots of displacement, velocity, and acceleration, we should review the relationship between these parameters. Figure 1 shows three plots of displacement, velocity, and acceleration for a simple harmonic wave.

The displacement, shown in red, is given by the expression $d = \sin(\Omega t)$, where the displacement amplitude, d, is a function of the time, t. Note that the displacement plot is zero at the beginning and that the slope at that point is positive. The displacement function then reaches a maximum where the slope is zero. After that, it again passes through zero (where the slope is negative) to reach a minimum (where the slope is again zero). Finally, the displacement plot returns to zero at the end of the cycle, at 360°. The maximum, minimum, and the zeros are marked on the plot with large dots.

If we draw another sinusoidal function through these values, we will generate a second graph. This second plot, shown in green, is the graph of the velocity. Mathematically, the velocity is the derivative of the displacement function and can be expressed as $v = \Omega\cos(\Omega t)$. Note that the velocity reaches a maximum 1/4 of a cycle (90°) before the displacement (the velocity leads the displacement by 90°).

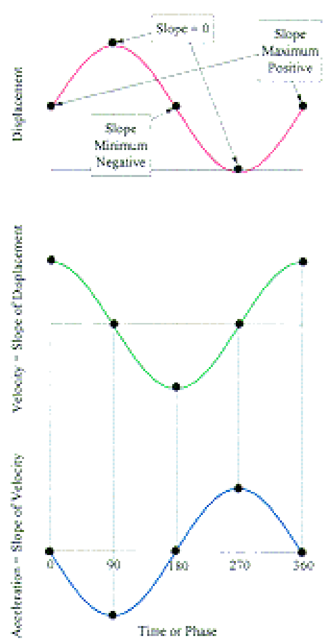

Figure 1

If we repeat this process (marking zeros, maxima, and minima) using the velocity plot, and plotting the slopes at these points, we will produce a plot of the acceleration, which is shown in Figure 1 in blue. The acceleration is the derivative of the velocity function and can be expressed as $a = -\Omega^2\sin(\Omega t)$. The acceleration is related to the velocity in the same way that the velocity is related to the displacement; that is, the acceleration plot reaches a maximum 1/4 of a cycle (90°) before the velocity. Because of this, the acceleration leads the displacement by 1/2 cycle or 180°.

Rotordynamic behavior

Vibration can be measured using displacement probes, velocity pickups, or accelerometers. Shaft relative displacement is nearly always used for machinery with sleeve and tilting pad bearings, but when housing measurements are useful, measurements may be made of either motion displacement, velocity, or acceleration. As discussed above, the basic relationship between these three parameters is relatively simple and can be found in basic textbooks and papers. The polar and Bode plots of displacement, velocity, and acceleration seem, however, to be very rare. Here they are.

Figure 2 shows a Bode plot of the synchronous (1X) lateral displacement (red), velocity (green), and acceleration (blue) of a rotor which is subject to a rotating unbalance (the amplitudes have been normalized on the amplitude part of the Bode plot to allow easy comparison). The phases of these parameters have identical shapes but are shifted

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from each other by 90°. Note also that, for each individual phase plot, the phase passes through 90° relative phase lag at the rotor undamped natural frequency, given by $\Omega_{90} = \sqrt{K/M}$.

The amplitude part of the Bode plot shows differences in behavior between the displacement (red), velocity (green), and acceleration (blue) at both the synchronous balance resonance and at high speed. When damping is present in the system, the peak of displacement amplitude occurs slightly later than the 90° phase shift. To be precise, the 90° phase shift occurs when $K - \Omega^2 M = 0$, or when the shaft speed, Ω_{90}, is equal to $\sqrt{K/M}$; the amplitude peak, obtained by the zero slope of the amplitude curve, is

$$\Omega_{res} = \sqrt{\frac{K}{M - \frac{D^2}{2K}}}$$

Note that the synchronous fluid damping multiplier, $(1-\lambda)$, is missing. This is because these are presumed to be housing measurements; shaft measurements of velocity and acceleration are nearly useless.

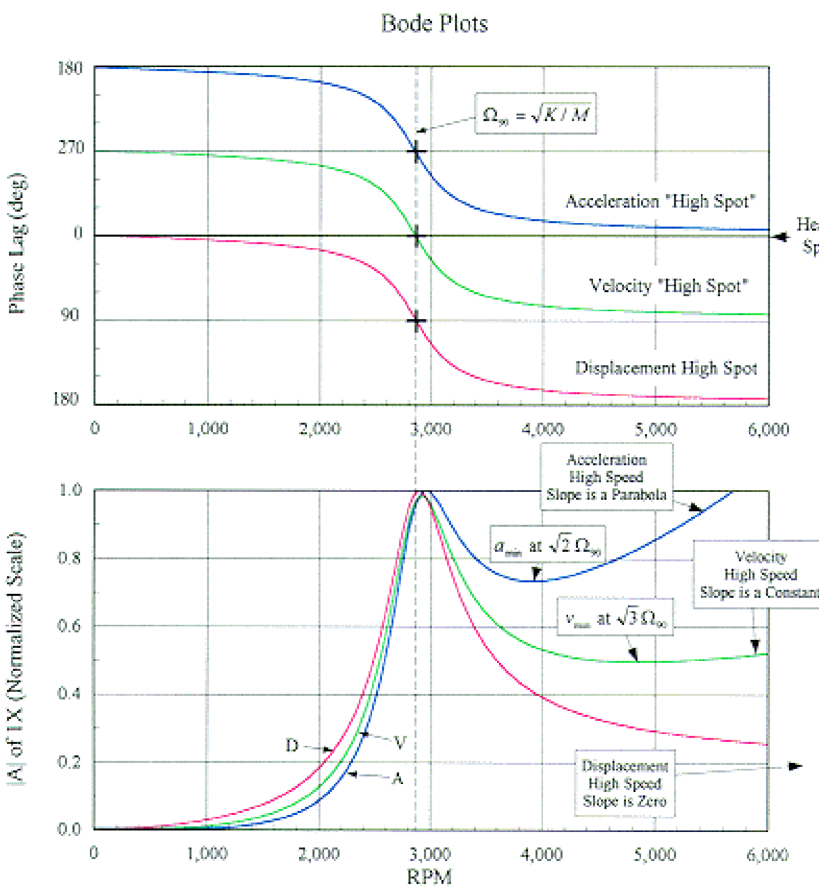

Figure 2

The velocity resonance occurs at a slightly higher speed than the displacement resonance. Similarly, the acceleration resonance occurs at a slightly higher speed than the velocity resonance.

Most dramatic is the difference in high speed behavior. At high speed, the amplitude of displacement of the rotor system eventually becomes constant, with zero slope. However, at high speed, the velocity amplitude has a slope which is a function of the first power of the rotative speed, that is $|v| = f(\Omega^1)$, tends to a sloped straight line, and will continue to increase with shaft speed. The acceleration amplitude, however, increases with a slope which is a function of the ***second*** power of the shaft speed, $|a| = f(\Omega^2)$, a parabola, and it increases rapidly with increasing shaft speed, approaching this parabola.

Note that the Bode plots of velocity and acceleration show minima occurring after the resonance. It can be shown that, for low damping, the velocity minimum occurs at a shaft speed approximately equal to $\sqrt{3}\ \Omega_{90}$, and the acceleration minimum occurs at a shaft speed approximately equal to $\sqrt{2}\ \Omega_{90}$. I believe that these very neat relationships have not been previously described. If you know of any prior publication of this relationship, please let me know. If you need a copy of my derivation, please request it.

This differing behavior causes changes in the appearance of a polar plot. Figure 3 shows a polar plot of the displacement (red), velocity (green), and acceleration (blue). The displacement shows a normal, circular behavior which begins moving in the direction of the unbalance at 0° (The Heavy Spot and the High Spot are together). At high speeds, above the resonance, the displacement polar moves in the direction of the origin, but never really gets there, approaching a fixed value of amplitude and phase.

The velocity polar begins 90° earlier than the displacement polar (at low speed the velocity "High Spot" leads the Heavy Spot by 90°) and moves in a circular pattern until it gets to its amplitude minimum. At that point, the velocity

polar moves in a more radial direction while slowly approaching the 90° axis.

The acceleration polar begins by moving in a direction 180° away from the unbalance force (at low speed the acceleration "High Spot" leads the Heavy Spot by 180°). After passing through the resonance, it takes off strongly in a direction almost, but not quite, parallel to the 0° axis. In fact, the acceleration polar actually diverges slightly from the 0° axis. This gives the acceleration polar the shape of an inverted question mark.

Interestingly, the velocity and acceleration minima discussed above in reference to the Bode plots are quite visible in the polar plots as corners and are marked with small circles.

It is important to note that the behaviors of the velocity and acceleration polar plots are very different from the displacement polar plot and can lead to confusion when attempting to balance a machine. At low speed, the velocity polar leads the displacement polar by 90°, while the acceleration polar leads the displacement polar by 180°. At low speed, the velocity polar moves at a right angle to the unbalance, the acceleration polar moves away from the unbalance, while the displacement polar moves toward the unbalance. Similarly, at high speed, the velocity polar lags the unbalance by about 90°, the acceleration polar has a phase angle in nearly the same direction as the unbalance, while the displacement polar has a phase angle about 180° opposite the unbalance. This can cause confusion if you are trying to balance using velocity or acceleration data. Of course, any time that you do not watch the slow roll bow (the bow at speeds well below a resonance) with shaft relative displacement probes, you have missed most of the vital data required for competent balancing.

If you observe the amplitude and phase of a simple linear system (such as a monkey on a bungee or a mass on a spring) as a function of frequency, you get polar and Bode plots. However, the rotating machine is very interesting because its phase angles not only occur in time, but in actual physical angles on the machine rotor itself.

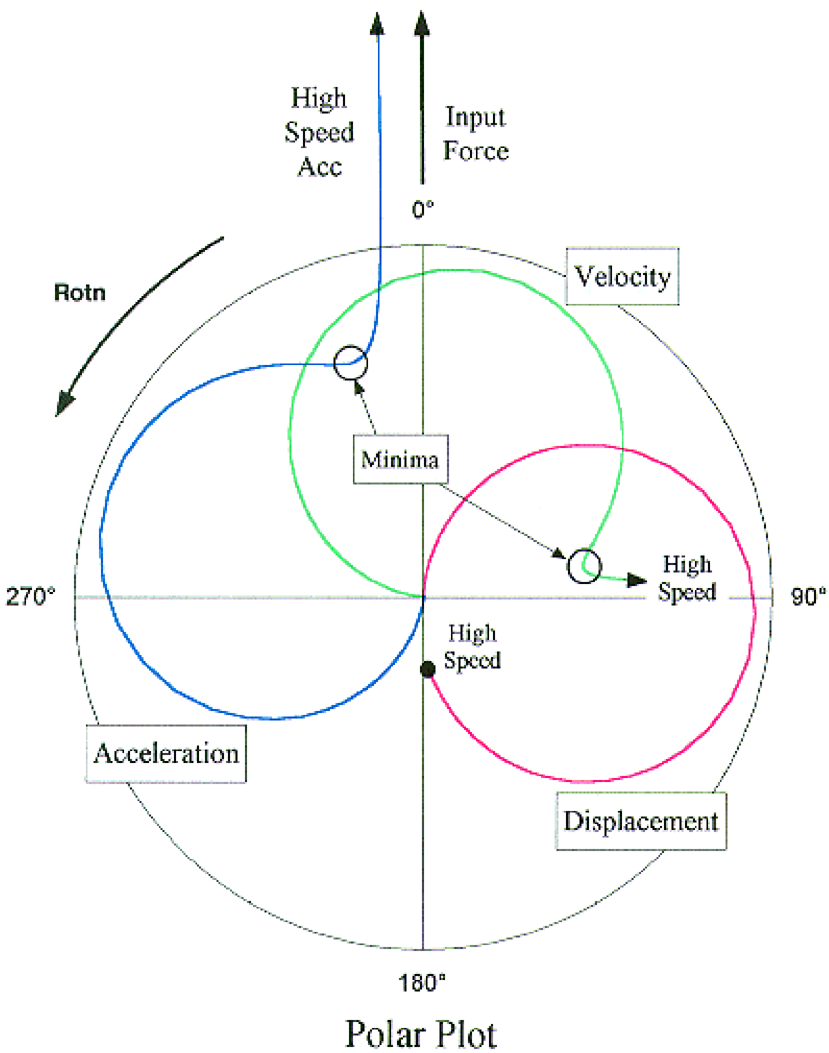

Figure 3

Addendum

When forced by a rotating unbalance (which follows an rpm square law), the displacement response of the fluid resonance has a behavior essentially similar to the velocity curve of mechanical resonance in Figure 2. This is because the fluid resonance amplitude is also a function of the first power of perturbation speed, instead of the square law behavior of the mechanical resonance. The minimum displacement amplitude, however, occurs at approximately twice (not $\sqrt{3}$ times) the fluid resonance speed. The fluid resonance is induced by the circumferential fluid flow in rotor-to-stator interfaces during nonsynchronous perturbation, and it also occurs at a nonsynchronous perturbation frequency of $\lambda\Omega$ ■.

If you know of any previous publication of the relationship between velocity minima and acceleration minima, please contact me by writing Orbit Editor, P.O. Box 157, Minden, NV 89448. A copy of my derivation is available upon request.

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